Strength of Bolted Joints between Fin Mounts and Fins for TRA Level 3 Certification Rocket
By: Ben Staal

From Shigley's Mechanical Engineering Design, Eighth Edition

Thickness of Fin Mount:

$t_{mount} := 0.125\ in$

$t_{mount} = 3.175\ mm$

Thickness of Fin:

$t_{fin} := 0.125\ in$

$t_{fin} = 3.175\ mm$

Using 1/4-20 Hex Bolts, 0.875in long:
Tensile Stress Area of Bolts (from Table 8-2 in Shigley's):

Nominal Thickness of Washers:

$t_{washer} := 0.043\ in$

$t_{washer} = 1.092\ mm$

The major diameter and tensile stress area of the threaded section of the bolt are:

$d_{major} := 0.25\ in$

$d_{major} = 6.35\ mm$

$A_{bolt} := 0.0318\ in^2$

$A_{bolt} = 20.516\ mm^2$

Grip Length is the total length from fastener head to start of nut

$l_{grip} := \left(2 \cdot t_{washer}\right) + \left(2 \cdot t_{mount}\right) + t_{fin}$

$l_{grip} = 0.461\ in$

$l_{grip} = 11.709\ mm$

The bolts are made from 18-8 (or 303) Stainless Steel
From MatWeb:

$E_{303} := 28000\ ksi$

$E_{303} = 193\ GPa$

Since the bolt is fully threaded, the stiffness of the bolt is (from Equation 8-16 in Shigley's):

$$k_b := \frac{\left(A_{bolt} \cdot E_{303}\right)}{l_{grip}}$$

$$k_b = \left(1.931 \cdot 10^{6}\right) \frac{lbf}{in}$$

$$k_b = \left(3.382 \cdot 10^{8}\right) \frac{N}{m}$$

To calculate the total stiffness of the washers, fin mounts and fins, the stiffness of each separate component must be calculated first.

The fin mounts are made from 5052-H32 Aluminum, therefore (from MatWeb):

$$E_{5052} := 10200\ ksi$$

$$E_{5052} = 70\ GPa$$

The holes through the fin mounts are:

$$d_{mount} := 0.266\ in$$

$$d_{mount} = 6.756\ mm$$

Since in most situations $\alpha = 30°$, the stiffness of fin mounts is equal to (from equation 8-20 in Shigley's):

$$k_{mount} := \frac{0.5774\ \pi \cdot E_{5052} \cdot d_{mount}}{2 \cdot \ln\left(5 \cdot \frac{\left(0.5774 \cdot t_{mount} + 0.5 \cdot d_{mount}\right)}{\left(0.5774 \cdot t_{mount} + 2.5 \cdot d_{mount}\right)}\right)}$$

$$k_{mount} = \left(7.446 \cdot 10^{6}\right) \frac{lbf}{in}$$

$$k_{mount} = \left(1.304 \cdot 10^{9}\right) \frac{N}{m}$$

The fins are made from G-10 Fiberglass Epoxy Laminate Sheet, therefore (from "Mechanical Properties of Glass Fiber Reinforced Polymers Members for Structural Applications" by Alexandre Landesmanna, Carlos Alexandre Serutia and Eduardo de Miranda Batistaa http://www.scielo.br/scielo.php?script=sci_arttext&pid=S1516-14392015000601372):

$$E_{G10} := 25.7\ GPa$$

$$E_{G10} = 3727\ ksi$$

$$k_{fin} := \frac{0.5774\ \pi \cdot E_{G10} \cdot d_{mount}}{2 \cdot \ln\left(5 \cdot \frac{(0.5774 \cdot t_{fin} + 0.5 \cdot d_{mount})}{(0.5774 \cdot t_{fin} + 2.5 \cdot d_{mount})}\right)}$$

$$k_{fin} = \left(2.721 \cdot 10^{6}\right) \frac{lbf}{in}$$

$$k_{fin} = \left(4.765 \cdot 10^{8}\right) \frac{N}{m}$$

To determine the total stiffness of the material, the spring rates need to be added in series

$$k_{series} := \frac{1}{\left(\frac{1}{k_{fin}} + \frac{2}{k_{mount}}\right)}$$

$$k_{series} = \left(1.572 \cdot 10^{6}\right) \frac{lbf}{in}$$

$$k_{series} = \left(2.753 \cdot 10^{8}\right) \frac{N}{m}$$

The bolts are tightened with a torque of:

$$T_{bolt} := 60\ lbf \cdot in$$

$$T_{bolt} = 6.779\ N \cdot m$$

From equation 8-27 in Shigley's, the torque results in a preload of:

$$F_{preload} := \frac{T_{bolt}}{0.2 \cdot d_{major}}$$

$$F_{preload} = \left(1.2 \cdot 10^{3}\right)\ lbf$$

$$F_{preload} = \left(5.338 \cdot 10^{3}\right)\ N$$

The preload stress on the bolt is:

$$\sigma_{preload} := \frac{F_{preload}}{A_{bolt}}$$

$$\sigma_{preload} = 37.736\ ksi$$

$$\sigma_{preload} = 260.18\ MPa$$

From McMaster's website for bolts, the tensile strength of the bolts is equal to:

$\sigma_{bolt\_strength} := 70\ ksi$

$\sigma_{bolt\_strength} = 483\ MPa$

Assume a worst case tensile load one a single bolt is equal to twice the weight of the rocket with a motor loaded.

$P_{load} := 2 \cdot 55\ lbf$

$P_{load} = 489.304\ N$

The portion of the load taken by the bolt: $P_{bolt}$

The portion of the load taken by the members: $P_{members}$

The fraction of the external load carried by the bolt: $C$

$$C := \frac{k_b}{k_b + k_{series}}$$

$C = 0.551$

$P_{bolt} := C \cdot P_{load}$

$P_{bolt} = 60.641\ lbf$

$P_{bolt} = 269.744\ N$

$P_{members} := P_{load} - P_{bolt}$

$P_{members} = 49.359\ lbf$

$P_{members} = 219.56\ N$

Resultant bolt load: $F_{bolt} := P_{bolt} + F_{preload}$

$F_{bolt} = \left(1.261 \cdot 10^3\right)\ lbf$

$F_{bolt} = \left(5.608 \cdot 10^3\right)\ N$

The resulting stress in the bolt:

$$\sigma_{bolt} := \frac{F_{bolt}}{A_{bolt}}$$

$$\sigma_{bolt} = 39.643\ ksi$$

$$\sigma_{bolt} = 273.327\ MPa$$

The tensile load safety factor is:

$$S_{bolt\_tensile} := \frac{\sigma_{bolt\_strength}}{\sigma_{bolt}}$$

$$S_{bolt\_tensile} = 1.766$$

Therefore, the bolt can handle 1.766 times the assumed worst case tensile load in addition to the preload from torquing the bolt to 60 $lbf \cdot in.$

The resultant load on the members is:

$$F_{members} := P_{members} - F_{preload}$$

$$F_{members} = -1.151 \cdot 10^{3}\ lbf$$

$$F_{members} = -5.118 \cdot 10^{3}\ N$$

The negative load indicates that the load is in the opposite direction as the bolt preload.

Using the load in the members, the stress in the fin mounts and fins can be determined.
Again, from MatWeb:

$$\sigma_{5052\_yield} := 28\ ksi$$

Since the yield strength of G10 fiberglass sheet is not listed, a value of 50% of the ulimate strength is used.

$$\sigma_{G10\_yield} := 0.50 \cdot 38\ ksi$$

The load is transferred to the mount via the face of the washer

$$OD_{washer} := 0.625\ in$$

$$OD_{washer} = 15.875\ mm$$

$ID_{washer} := 0.281\ in$

$ID_{washer} = 7.137\ mm$

The area of the washer face is:

$$A_{washer} := \left(\pi \cdot \left(\frac{OD_{washer}}{2}\right)^2\right) - \left(\pi \cdot \left(\frac{ID_{washer}}{2}\right)^2\right)$$

$A_{washer} = 0.245\ in^2$

$A_{washer} = 157.922\ mm^2$

Therefore, the resulting tensile stresses under maximum load and the bolt preload are:

$$\sigma_{members} := \frac{F_{members}}{A_{washer}}$$

$\sigma_{members} = -4.701\ ksi$

$\sigma_{members} = -32.41\ MPa$

The safety factor in the fin mounts is:

$$S_{mount\_tensile} := \left|\frac{\sigma_{5052\_yield}}{\sigma_{members}}\right|$$

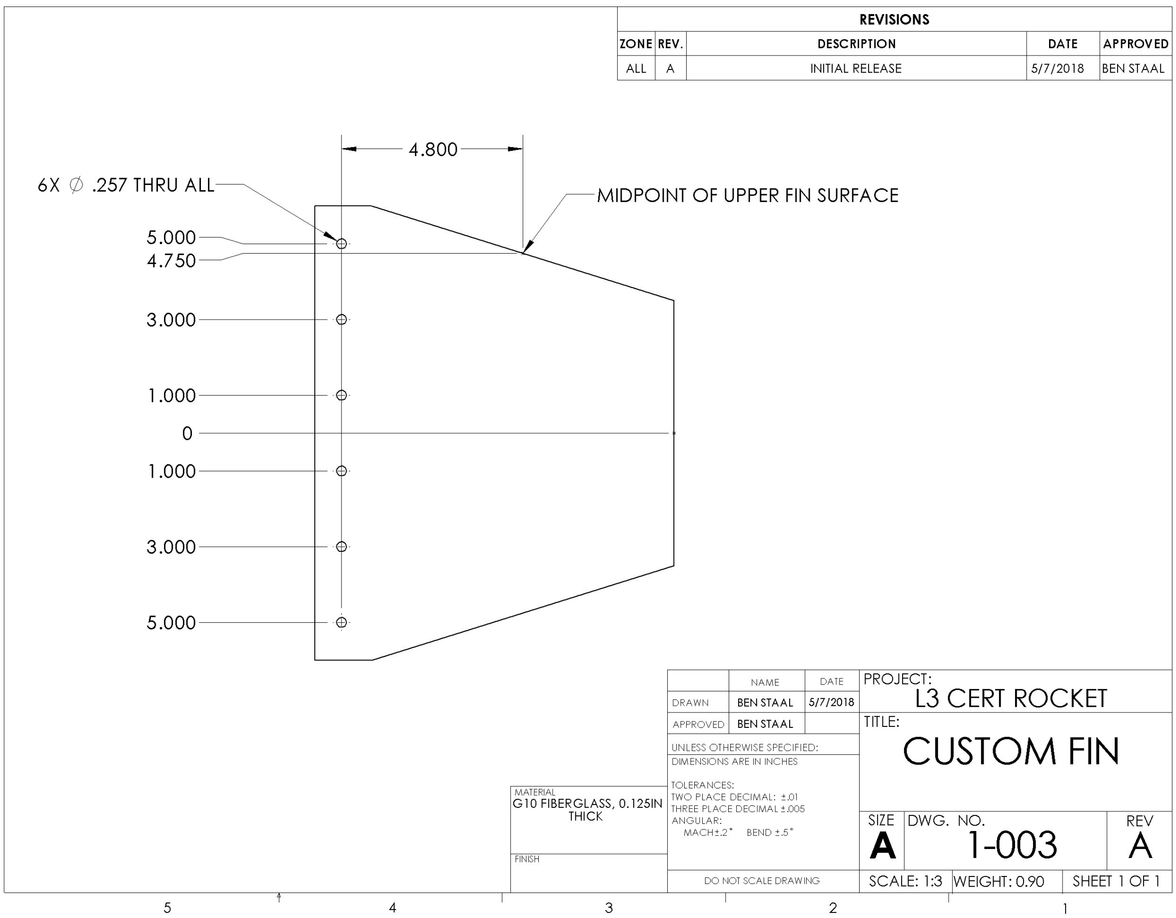
$S_{mount\_tensile} = 5.957$

$$S_{fin\_tensile} := \left|\frac{\sigma_{G10\_yield}}{\sigma_{members}}\right|$$

$S_{fin\_tensile} = 4.042$

The predominant stress on the fins and fins mounts during flight operations is shear stress.

Since the primary shear load during flight will be caused by drag on the fin, the drag load can be simplified to a point load at the centroid of the top face with a total force equal to that of the entire expected drag force.




To determine the worst case drag force, it is assumed that the rocket will be flying at the critical fin flutter speed (calculated previously) at sea level and standard temperature and pressure. This will lead to an exagerated drage since the density of the fluid will be higher than at the launch site and at a speed faster than the rocket is designed to achieve.

From the Engineering Toolbox:

$$c_d := 1.98$$

$$\rho := 1.2 \frac{kg}{m^3}$$

$$\rho = 0.075 \frac{lbm}{ft^3}$$

$A_{fin} := 8.0\ in \cdot 0.125\ in$

$A_{fin} = 1\ in^2$

$A_{fin} = 645.16\ mm^2$

$v_{critical} := 700\ mph$

$v_{critical} = 312.928\ \frac{m}{s}$

$F_{drag} := -c_d \cdot \left(\frac{1}{2}\right) \cdot \rho \cdot {v_{critical}}^2 \cdot A_{fin}$

$F_{drag} = -16.873\ lbf$

$F_{drag} = -75.054\ N$

Since the fin is static with respect to the rocket's frame of reference
The resulting shear reaction V:

$V := -F_{drag}$

The resulting moment reaction M:

$M := F_{drag} \cdot 4.800\ in$

$M = -80.989\ lbf \cdot in$

$M = -9.151\ N \cdot m$

Since there are six bolts per fin, the primary shear per bolt is:

$n_{bolts} := 6$

$F' := \frac{V}{n_{bolts}}$

$F' = 2.812\ lbf$

$F' = 12.509\ N$

To calculate the resultant moments on each bolt, the moment arm for each must be determined:

$$r := \begin{bmatrix} 5 \\ 3 \\ 1 \\ -1 \\ -3 \\ -5 \end{bmatrix} in$$

$$r = \begin{bmatrix} 127 \\ 76.2 \\ 25.4 \\ -25.4 \\ -76.2 \\ -127 \end{bmatrix} mm$$

The moment load or secondary shear at each bolt can be found with the following equation (8-54 from Shigley's):

$$F'' := \frac{M \cdot r}{{r_0}^2 + {r_1}^2 + {r_2}^2 + {r_3}^2 + {r_4}^2 + {r_5}^2}$$

$$F'' = \begin{bmatrix} -5.785 \\ -3.471 \\ -1.157 \\ 1.157 \\ 3.471 \\ 5.785 \end{bmatrix} lbf$$

$$F'' = \begin{bmatrix} -25.733 \\ -15.44 \\ -5.147 \\ 5.147 \\ 15.44 \\ 25.733 \end{bmatrix} N$$

The resulting force at each bolt is:

$$F := F'' + F'$$

$$F = \begin{bmatrix} -2.973 \\ -0.659 \\ 1.655 \\ 3.969 \\ 6.283 \\ 8.597 \end{bmatrix} lbf$$

$$F = \begin{bmatrix} -13.224 \\ -2.931 \\ 7.362 \\ 17.656 \\ 27.949 \\ 38.242 \end{bmatrix} N$$

Since the bolts connecting the fin to the fin mounts are fully threaded, the shear stress in each bolt is:

From Table 8-2 in Shigley's, for a 1/4-20 bolt:

$A_s := 0.0269\ in^2$

$\tau_{bolts} := \dfrac{F}{A_s}$

$$\tau_{bolts} = \begin{bmatrix} -110.514 \\ -24.492 \\ 61.529 \\ 147.551 \\ 233.572 \\ 319.594 \end{bmatrix} psi$$

$$\tau_{bolts} = \begin{bmatrix} -0.762 \\ -0.169 \\ 0.424 \\ 1.017 \\ 1.61 \\ 2.204 \end{bmatrix} MPa$$

From Shigley's, the shear strength of the bolts can be approximated to be $0.577 \cdot \sigma_{bolt\_strength}$

$\tau_{303} := 0.577 \cdot \sigma_{bolt\_strength}$

$\tau_{303} = 40.39\ ksi$

$\tau_{303} = 278.479\ MPa$

Therefore, the safety factor in the bolts to aerodynamic shear loading is:

$S_{shear\_bolts} := \dfrac{\tau_{303}}{\tau_{bolts}}$

$$S_{shear\_bolts} = \begin{bmatrix} -365.475 \\ -1.649 \cdot 10^3 \\ 656.435 \\ 273.736 \\ 172.923 \\ 126.379 \end{bmatrix}$$

The bearing stress in the fins is:

$$A_{fin\_bearing} := t_{fin} \cdot d_{major}$$

$$\sigma_{fin\_bearing} := \frac{F}{t_{fin} \cdot d_{major}}$$

$$\sigma_{fin\_bearing} = \begin{bmatrix} -95.13 \\ -21.083 \\ 52.964 \\ 127.012 \\ 201.059 \\ 275.106 \end{bmatrix} psi$$

$$\sigma_{fin\_bearing} = \begin{bmatrix} -655.9 \\ -145.362 \\ 365.177 \\ 875.715 \\ 1.386 \cdot 10^{3} \\ 1.897 \cdot 10^{3} \end{bmatrix} kPa$$

Therefore, the minimum safety factor in the fins to aerodynamic drag is:

$$S_{fins\_bearing} := \frac{\sigma_{G10\_yield}}{\sigma_{fin\_bearing_5}}$$

$$S_{fins\_bearing} = 69.064$$